Exercise 19C: Solutions

1
$$\begin{aligned} s &= 30, u = 0, a = 1.5 \\ s &= ut + \frac{1}{2}at^2 \\ 30 &= \frac{1}{2} \times 1.5 \times t^2 \\ t^2 &= 40 \\ t &= \sqrt{40} \\ &= 2\sqrt{10} \text{ s} \end{aligned}$$

2
$$\begin{aligned} u &= 25, v = 0, t = 3 \\ s &= \frac{1}{2}(u + v)t \\ &= \frac{1}{2}(25 + 0) \times 3 \\ &= 37.5 \text{ m} \end{aligned}$$

3 a For constant acceleration,

$$\begin{aligned} \text{acceleration} &= \frac{\text{change in velocity}}{\text{change in time}} \\ &= \frac{27}{9} = 3 \text{ m/s}^2 \end{aligned}$$

b
$$\begin{aligned} u &= 30, v = 50, a = 3 \\ v &= u + at \\ 50 &= 30 + 3t \\ 3t &= 20 \\ t &= \frac{20}{3} = 6\frac{2}{3} \text{ s} \end{aligned}$$

c
$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ &= \frac{1}{2} \times 3 \times 15^2 \\ &= 337.5 \text{ m} \end{aligned}$$

d
$$\begin{aligned} 200 \text{ km/h} &= 200 \div 3.6 \\ &= \frac{500}{9} \text{ m/s} \\ u &= 0, v = \frac{500}{9}, a = 3 \\ v &= u + at \\ \frac{500}{9} &= 0 + 3t \\ 3t &= \frac{500}{9} \\ t &= \frac{500}{27} \\ &= 18\frac{14}{27} \text{ s} \end{aligned}$$

4 a
$$\begin{aligned} 45 \text{ km/h} &= 45 \div 3.6 \\ &= 12.5 \text{ m/s} \end{aligned}$$

For constant acceleration,

$$\begin{aligned} \text{acceleration} &= \frac{\text{change in velocity}}{\text{change in time}} \\ &= \frac{12.5}{5} = 2.5 \text{ m/s}^2 \end{aligned}$$

b $s = ut + \frac{1}{2}at^2$

$= \frac{1}{2} \times 2.5 \times 5^2$

$= 31.25 \text{ m}$

5 **a** $90 \text{ km/h} = 90 \div 3.6$

$= 25 \text{ m/s}$

$u = 0, v = 25, a = 0.5$

$v = u + at$

$25 = 0 + 0.5t$

$0.5t = 25$

$t = \frac{2.5}{05} = 50 \text{ s}$

b $s = ut + \frac{1}{2}at^2$

$= \frac{1}{2} \times 0.5 \times 50^2$

$= 625 \text{ m}$

6 **a** $54 \text{ km/h} = 54 \div 3.6$

$= 15 \text{ m/s}$

$u = 15, a = -0.25, s = 250$

$s = ut + \frac{1}{2}at^2$

$250 = 15t + \frac{1}{2} \times -0.25t^2$

Multiply both sides by 8:

$2000 = 120t - t^2$

$t^2 - 120t + 2000 = 0$

$(t - 20)(t - 100) = 0$

$t = 100$ represents the train changing velocity and retuming to this point.

$\therefore \quad t = 20 \text{ s}$

b $v = u + at$

$= 15 + -0.25 \times 20$

$= 10 \text{ m/s}$

$= 10 \times 3.6 = 36 \text{ km/h}$

7 **a** $v = u + at$

$= 20 + -9.8 \times 4$

$= -19.2 \text{ m/s}$

b $s = ut + \frac{1}{2}at^2$

$= 20 \times 4 + \frac{1}{2} \times -9.8 \times 4^2$

$= 1.6 \text{ m}$

8 **a** $v = u + at$

$= -20 + -9.8 \times 4$

$= -59.2 \text{ m/s}$

b $s = ut + \frac{1}{2}at^2$

$= -20 \times 4 + \frac{1}{2} \times -9.8 \times 4^2$
$= -158.4 \text{ m}$

9 a $u = 49, s = 0, a = -9.8$

$s = ut + \frac{1}{2}at^2$
$0 = 49t + \frac{1}{2} \times -9.8 \times t^2$
$0 = 49t - 4.9t^2$
$0 = 4.9t(10 - t)$
$t = 10 \text{ s}$

b $u = 49, s = 102.9, a = -9.8$

$s = ut + \frac{1}{2}at^2$
$102.9 = 49t + \frac{1}{2} \times -9.8 \times t^2$
$102.9 = 49t - 4.9t^2$
$0 = 4.9t^2 - 49t + 102.9$

Divide by 4.9:

$t^2 - 10t + 21 = 0$
$(t - 3)(t - 7) = 0$

At both 3 s (going up) and 7 s (going down).

10a $v = u + at$
$= 4.9 - 9.8t$
$= 4.9(1 - 2t)$

b $s = ut + \frac{1}{2}at^2$
$= 4.9t + \frac{1}{2} \times -9.8 \times t^2$
$= 4.9t - 4.9t^2$
$= 4.9t(1 - t)\text{m/s}$

This is his displacement from the initial 3 m height.

$\therefore h = 4.9t(1 - t) + 3 \text{ m}$

c From part a, the diver's velocity is zero when

$4.9(1 - 2t) = 0$
$t = \frac{1}{2} = 0.5$

The maximum height reached is

$h = 4.9(0.5)(1 - 0.5) + 3$
$= 4.9 \times 0.25 + 3$
$= 4.225$

d The diver reaches the water when $h = 0$, so:

$4.9t(1 - t) + 3 = 0$
$49t - 49t^2 + 30 = 0$
$49t^2 - 49t - 30 = 0$
$(7t + 3)(7t - 10) = 0$
$t = \frac{10}{7} \text{ s}$

Since $t > 0$

11a Maximum height occurs when $v = 0$.

$$u = 19.6, a = -9.8, v = 0$$
$$v = u + at$$
$$0 = 19.6 - 9.8t$$
$$t = \frac{19.6}{98} = 2 \text{ s}$$

b
$$s = ut + \frac{1}{2}at^2$$
$$= 19.6 \times 2 + \frac{1}{2} \times -9.8 \times 2^2$$
$$= 19.6 \text{ m}$$

So the maximum height from the foot of the cliff is $19.6 + 24.5 = 44.1 \text{ m}$.

c
$$u = 19.6, s = 0, a = -9.8$$
$$s = ut + \frac{1}{2}at^2$$
$$0 = 19.6t + \frac{1}{2} \times -9.8 \times t^2$$
$$0 = 19.6t - 4.9t^2$$
$$0 = 4.9t(4 - t)$$
$$t = 4 \text{ s}$$

d
$$u = 19.6, s = -24.5, a = -9.8$$
$$s = ut + \frac{1}{2}at^2$$
$$-24.5 = 19.6t + \frac{1}{2} \times -9.8 \times t^2$$
$$-24.5 = 19.6t - 4.9t^2$$
$$0 = 4.9t^2 - 19.6t - 24.5$$

Divide by 4.9:

$$t^2 - 4t - 5 = 0$$
$$(t - 5)(t + 1) = 0$$
$$t = 5 \text{ s}$$

12 Let the distance between P and Q be x m.

$$u = 20, v = 40, s = x$$
$$v^2 = u^2 + 2as$$
$$1600 = 400 + 2ax$$
$$2ax = 1200$$
$$a = \frac{1200}{2x}$$
$$= \frac{600}{x}$$

At the halfway mark,

$$u = 20, a = \frac{600}{x}, s = \frac{x}{2}$$
$$v^2 = u^2 + 2as$$
$$= 400 + 2 \times \frac{600}{x} \times \frac{x}{2}$$
$$= 1000$$
$$v = \sqrt{1000}$$
$$= 10\sqrt{10} \text{ m/s}$$